Contents

Preface		vii
Chapter 1. Background		
	Notational and Terminological Conventions	1
2.	Prerequisites	2
3.	Functions	2
4.	Vectors	4
5.	Differential Equations	7
6.	Notation for Sets	8
7.	Real-Variable Theory	9
8.	Function Spaces	10
\mathbf{Ch}	apter 2. The Equations of Motion for	
	Extensible Strings	13
1.	Introduction	13
	The Classical Equations of Motion	14
	The Linear Impulse-Momentum Law	25
4.	The Equivalence of the Linear Impulse-Momentum	
	Law with the Principle of Virtual Power	28
	Jump Conditions	31
	The Existence of a Straight Equilibrium State	33
	Purely Transverse Motions	35
	Perturbation Methods and the Linear Wave Equation	37
	The Justification of Perturbation Methods	43
	Variational Characterization of the Equations	46
11.	Discretization	11
Ch	apter 3. Elementary Problems for Elastic Strings	53
	Introduction	53
	Equilibrium of Strings Under Vertical Loads	54
	The Catenary Problem	57
	The Suspension Bridge Problem	69
	Equilibrium of Strings under Normal Loads	71
	Equilibrium of Strings Under Central Forces	79
	Circular States of a Spinning String	82
	Travelling Waves	83
	Radial Oscillations	85
	Combined Whirling and Radial Motions	86
11.	Massless Springs	86

xiv CONTENTS

12.	Comments and Historical Notes	92	
Chapter 4. Planar Steady-State Problems			
	for Elastic Rods	93	
1.	Formulation of the Governing Equations	93	
	Planar Equilibrium States of Straight Rods		
	under Terminal Loads	104	
3.	Equilibrium of Rings under Hydrostatic Pressure	109	
	Asymptotic Shape of Inflated Rings	118	
	Straight Configurations of a Whirling Rod	124	
6.	Simultaneous Whirling and Breathing Oscillations		
	of a Ring	131	
7.	Bibliographical Notes	132	
$\mathbf{C}\mathbf{h}$	apter 5. Introduction to Bifurcation Theory		
	and its Applications to Elasticity	133	
1.	The Simplest Buckling Problem	133	
	Classical Buckling Problems of Elasticity	139	
	Mathematical Concepts and Examples	146	
	Basic Theorems of Bifurcation Theory	157	
	Applications of the Basic Theorems		
	to the Simplest Buckling Problem	165	
6.	Perturbation Methods	171	
7.	Dynamics and Stability	176	
	Bibliographical Notes	179	
\mathbf{Ch}	apter 6. Global Bifurcation Problems		
Ch	apter 6. Global Bifurcation Problems for Strings and Rods	179	
	for Strings and Rods	179 179	
1.	-	179	
1. 2.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem		
1. 2. 3.	for Strings and Rods The Equations for the Steady Whirling of Strings	179 182	
1. 2. 3. 4.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings	179 182 191	
1. 2. 3. 4. 5.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings	179 182 191 196	
1. 2. 3. 4. 5.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory	179 182 191 196	
1. 2. 3. 4. 5. 6.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity	179 182 191 196 202	
1. 2. 3. 4. 5. 6.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory	179 182 191 196 202	
1. 2. 3. 4. 5. 6.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions	179 182 191 196 202 205 209	
1. 2. 3. 4. 5. 6. 7. 8. 9.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions Planar Buckling of Rods. Nonbifurcating Branches	179 182 191 196 202 205 209 212	
1. 2. 3. 4. 5. 6. 7. 8. 9.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions Planar Buckling of Rods. Nonbifurcating Branches Global Disposition of Solution Sheets	179 182 191 196 202 205 209 212 214	
1. 2. 3. 4. 5. 6. 7. 8. 9. 10.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions Planar Buckling of Rods. Nonbifurcating Branches Global Disposition of Solution Sheets Other Planar Buckling Problems for Straight Rods	179 182 191 196 202 205 209 212 214 219	
1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions Planar Buckling of Rods. Nonbifurcating Branches Global Disposition of Solution Sheets Other Planar Buckling Problems for Straight Rods Follower Loads	179 182 191 196 202 205 209 212 214 219 221	
1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions Planar Buckling of Rods. Nonbifurcating Branches Global Disposition of Solution Sheets Other Planar Buckling Problems for Straight Rods Follower Loads Buckling of Arches	179 182 191 196 202 205 209 212 214 219 221 223	
1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. Ch	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions Planar Buckling of Rods. Nonbifurcating Branches Global Disposition of Solution Sheets Other Planar Buckling Problems for Straight Rods Follower Loads Buckling of Arches Buckling of Whirling Rods	179 182 191 196 202 205 209 212 214 219 221 223 227	
1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. Ch	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions Planar Buckling of Rods. Nonbifurcating Branches Global Disposition of Solution Sheets Other Planar Buckling Problems for Straight Rods Follower Loads Buckling of Arches Buckling of Whirling Rods apter 7. Variational Methods	179 182 191 196 202 205 209 212 214 219 221 223 227 231	
1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. Ch 1. 2.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions Planar Buckling of Rods. Nonbifurcating Branches Global Disposition of Solution Sheets Other Planar Buckling Problems for Straight Rods Follower Loads Buckling of Arches Buckling of Whirling Rods apter 7. Variational Methods Introduction	179 182 191 196 202 205 209 212 214 219 221 223 227 231	
1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. Ch 1. 2. 3.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions Planar Buckling of Rods. Nonbifurcating Branches Global Disposition of Solution Sheets Other Planar Buckling Problems for Straight Rods Follower Loads Buckling of Arches Buckling of Whirling Rods apter 7. Variational Methods Introduction The Multiplier Rule	179 182 191 196 202 205 209 212 214 219 221 223 227 231 231 234	
1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. Ch 1. 2. 3. 4.	for Strings and Rods The Equations for the Steady Whirling of Strings Kolodner's Problem Other Problems for Whirling Strings The Drawing and Whirling of Strings Planar Buckling of Rods. Global Theory Planar Buckling of Rods. Imperfection Sensitivity via Singularity Theory Planar Buckling of Rods. Constitutive Assumptions Planar Buckling of Rods. Nonbifurcating Branches Global Disposition of Solution Sheets Other Planar Buckling Problems for Straight Rods Follower Loads Buckling of Arches Buckling of Whirling Rods apter 7. Variational Methods Introduction The Multiplier Rule Direct Methods	179 182 191 196 202 205 209 212 214 219 221 223 227 231 234 238	

CONTENTS xv

7.	The Second Variation. Bifurcation Problems	257
8.	Notes	260
\mathbf{Ch}	apter 8. Theory of Rods Deforming in Space	261
	Introduction	261
2.	Outline of the Essential Theory	262
	The Exact Equations of Motion	268
	The Equations of Constrained Motion	272
	The Use of Exact Momenta	275
6.	The Strains and the Strain-Rates	276
7.	The Preservation of Orientation	279
8.	Constitutive Equations Invariant	
	under Rigid Motions	284
9.	Invariant Dissipative Mechanisms	
	for Numerical Schemes	289
10.	Monotonicity and Growth Conditions	294
11.	Transverse Hemitropy and Isotropy	300
12.	Uniform Rods. Singular Problems	309
13.	Representations for the Directors in Terms	
	of Euler Angles	311
14.	Boundary Conditions	313
15.	Impulse-Momentum Laws and the Principle	
	of Virtual Power	319
16.	Hamilton's Principle for Hyperelastic Rods	323
17.	Material Constraints	324
	Planar Motions	327
	Classical Theories	329
	General Theories of Cosserat Rods	332
21.	Historical and Bibliographical Notes	333
\mathbf{Ch}	apter 9. Spatial Problems for Rods	335
	Summary of the Governing Equations	335
2.	Kirchhoff's Problem for Helical Equilibrium States	337
3.	General Solutions for Equlibria	340
4.	Travelling Waves in Straight Rods	344
5.	Buckling under Terminal Thrust and Torque	347
6.	Lateral Instability	349
$\mathbf{C}\mathbf{h}$	apter 10. Axisymmetric Equilibria of Shells	353
	Formulation of the Governing Equations	353
	Buckling of a Transversely Isotropic Circular Plate	359
3.		
	of Aeolotropic Circular Plates	366
4.	Buckling of Aeolotropic Plates	373
	Buckling of Spherical Shells	375
	Buckling of Cylindrical Shells	379
	Asymptotic Shape of Inflated Shells	380
	Membranes	381
9.	Eversion	383

xvi CONTENTS

\mathbf{Ch}	apter 11. Tensors	389
1.	Tensor Algebra	389
2.	Tensor Calculus	398
3.	Indicial Notation	404
\mathbf{Ch}	apter 12. 3-Dimensional Continuum	
	Mechanics	407
1.	Kinematics	407
2.	Strain	409
3.	Compatibility	413
4.	Rotation	416
5.	Examples	418
6.	Mass and Density	422
7.	Stress and the Equations of Motion	423
8.	Boundary and Initial Conditions	430
9.	Impulse-Momentum Laws and	
	the Principle of Virtual Power	433
10.	Constitutive Equations of Mechanics	439
11.	Invariance under Rigid Motions	441
	Material Constraints	445
	Isotropy	458
	Thermomechanics	463
15.	The Spatial Formulation	473
\mathbf{Ch}	apter 13. 3-Dimensional Theory of Nonlinear	
	Elasticity	479
	Summary of the Governing Equations	479
	Constitutive Restrictions	481
	Order-Preservation: Monotonicity and Ellipticity	482
	Growth Conditions	490
	Special Constitutive Equations	491
	Existence and Regularity	492
	Versions of the Euler-Lagrange Equations	493
	Linear Elasticity	497
9.	Viscous Dissipation	499
	apter 14. Problems in Nonlinear Elasticity	501
1.	Elementary Static Problems in Cartesian Coordinates	501
	Torsion, Extension, Inflation, and Shear	
	of an Annular Sector	503
3.	Torsion and Related Equilibrium Problems	
	for Incompressible Bodies	507
4.	Torsion, Extension, Inflation, and Shear	
_	of a Compressible Annular Sector	511
	Flexure, Extension, and Shear of a Block	514
	Flexure, Extension, and Shear of a Compressible Block	516
	Dilatation, Cavitation, Inflation, and Eversion	522
	Other Semi-Inverse Problems	537
9.	Universal and Non-Universal Deformations	540

CONTENTS xvii

10. Antiplane Problems	543
11. Perturbation Methods	545
12. Instability of an Incompressible Body under Constant	
Normal Traction	556
13. Radial Motions of an Incompressible Tube	560
14. Universal Motions of Incompressible Bodies	562
15. Standing Shear Waves in an Incompressible Layer	567
16. Commentary. Other Problems	570
Chapter 15. Large-Strain Plasticity	571
1. Constitutive Equations	571
2. Refinements and Generalizations	577
3. Example: Longitudinal Motion of a Bar	579
4. Antiplane Shearing Motions	581
5. Notes	585
	587
Chapter 16. General Theories of Rods 1. Introduction	587
2. Curvilinear Coordinates	589
3. Geometry of Rod-like Bodies	592
4. Exact Equations of Motion	593
5. Semi-Intrinsic Theories	594
6. Induced Theories of Rods	595
7. Convergence	604
8. Rods with Two Directors	607
9. Elastic Rods	615
10. Elastic Rods with a Plane of Symmetry	619
11. Necking	624
12. The Treatment of Incompressibility	628
13. Intrinsic Theories of Rods	636
14. Mielke's Treatment of St. Venant's Principle	638
Chapter 17. General Theories of Shells	643
 Induced Shell Theories Shells with One Director 	643
	647 653
3. Drawing and Twisting of an Elastic Plate4. Axisymmetric Motions of Axisymmetric Shells	657
5. Global Buckled States of a Cosserat Plate	
	663
6. Thickness parameter. Eversion 7. The Treatment of Incompressibility.	666
7. The Treatment of Incompressibility	668
8. Intrinsic Theory of Special Cosserat Shells	669
9. Membranes	680
10. Asymptotic Methods. The von Kármán Equations	682
11. Justification of Shell Theories as Asymptotic Limits	687
12. Commentary. Historical Notes	690
Chapter 18. Dynamical Problems	693
1. The 1-Dimensional Quasilinear Wave Equation	693
2. The Riemann Problem. Uniqueness and	
Admissibility of Weak Solutions	697

xviii CONTENTS

3. Shearing Motions of Viscoplastic Layers	704
4. Dissipative Mechanisms and the Bounds They Induce	705
5. Shock Structure. Admissibility and Travelling Waves	715
6. Travelling Shear Waves in Viscoelatic Media	720
7. Blow-up in Three-Dimensional Hyperelasticity	727
Chapter 19. Appendix. Topics in Linear Analysis	733
1. Banach Spaces	733
2. Linear Operators and Linear Equations	736
Chapter 20. Appendix. Local Nonlinear Analysis 1. The Contraction Mapping Principle and the Implicit	741
Function Theorem 2. The Lyapunov-Schmidt Method. The	741
Poincaré Shooting Method	744
Chapter 21. Appendix. Degree Theory and its	
Applications	747
1. Definition of the Brouwer Degree	747
2. Properties of the Brouwer Degree	751
3. Leray-Schauder Degree	757
4. One-Parameter Global Bifurcation Theorem	761
References	763
Index	801